Computer Science CSCI 355

Digital Logic and Computer Organization

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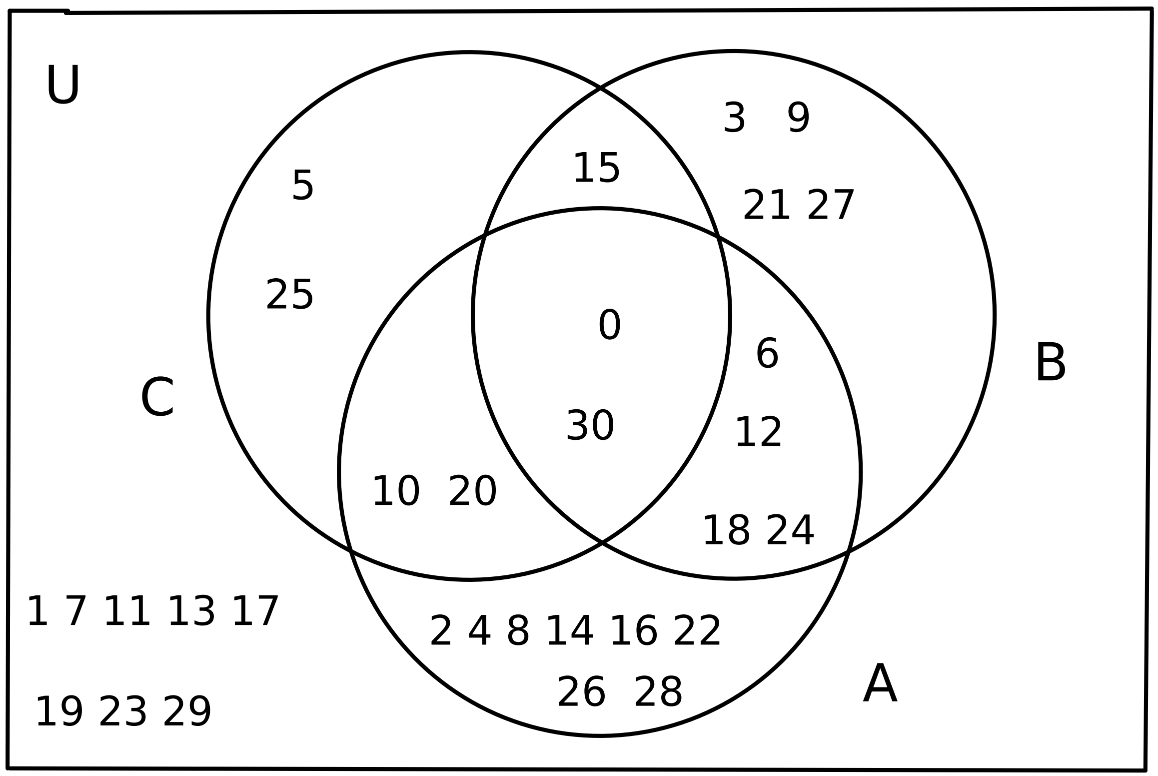
Set Example

Consider the set of integers from 0 to 30 inclusive where U represents the universe (all 31 integers) and ϕ represents the empty set.

- let A be the set of all multiples of 2 in U
- let B be the set of all multiples of 3 in U
- let C be the set of all multiples of 5 in U

Set Example cont.



Properties

- Closure
  - $\forall x, y \subseteq U, \quad x \cup y \subseteq U$ and $x \cap y \subseteq U$
    e.g. $A \cup B \subseteq U$ and $A \cap B \subseteq U$

- Identity
  - $\forall x \subseteq U, \quad x \cup \phi = x$ and $x \cap U = x$
    e.g. $A \cup \phi = A$ and $A \cap U = A$

- Commutativity
  - $\forall x, y \subseteq U, \quad x \cup y = y \cup x$ and $x \cap y = y \cap x$
    e.g. $A \cup B = B \cup A$ and $A \cap B = B \cap A$

Properties cont.

- Distributivity
  - $\forall x, y, z \subseteq U, \quad x \cap (y \cup z) = (x \cap y) \cup (x \cap z)$
    $\forall x, y, z \subseteq U, \quad x \cup (y \cap z) = (x \cup y) \cap (x \cup z)$
    e.g. $A \cup (B \cap C) = A \cup \{15\} = (A \cup B) \cap (A \cup C)$

- Complement
  - $\forall x \subseteq U, \quad \overline{x} = U - x \subseteq U, \; x \cup \overline{x} = U$ and $x \cap \overline{x} = \phi$
    e.g. $A \cap \overline{A} = \phi \;\; A \cup \overline{A} = U$

Boolean Algebra

- George Boole (1854)
  - introduced a systematic treatment of logic
- Huntington (1904)
  - defined Boolean algebra by providing 6 postulates that must be satisfied viz.,
    (1) Closure (2) Identity (3) Commutativity
    (4) Distributivity (5) Complement
    (6) Distinct Elements ($|U| >= 2$)
- Shannon (1938)
  - applied Boolean algebra to relay circuitry found in telephone routing switches
    Distinct Elements ($|U| = 2$)

Switching Algebra

- Two Valued Boolean Algebra
  - defined on a set B with two elements and two binary operators $+$ (OR) and . (AND) which satisfy Huntington's Postulates

Switching Algebra cont.

B = {0, 1}

x	y	x.y	x+y
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

Switching Algebra cont.

- Postulate 1 Closure
- Postulate 2 Identity
  - $1 + 0 = 1 \quad 0 + 0 = 0 \quad (x + 0 = x)$
    $1.1 = 1 \quad 0.1 = 0 \quad (x.1 = x)$
- Postulate 3 Commutativity
  - table rows are symmetric i.e.,
  - 01 row is the same as the 10 row
- Postulate 4 Distributivity
  - use Perfect Induction (truth table) to establish
- Postulate 5 Complement
  - $0 + \overline{0} = 0 + 1 = 1 \quad 1 + \overline{1} = 1 + 0 = 1 \quad (x + \overline{x} = 1)$
    $1.\overline{1} = 1.0 = 0 \quad 0.\overline{0} = 0.1 = 0 \quad (x.\overline{x} = 0)$
    0 and 1 are the complements of each other

Postulates, Theorems and Axioms

- Postulates
  - fundamental building blocks, assumed to be true
- Theorems
  - inferred from postulates, must be proven
- Axioms
  - small theorems

Proofs (Expression Equivalence)

- Perfect Induction
  - construct truth tables (literal analysis)
- Axiomatic Proof
  - apply Huntington's Postulates
    or other Theorems and/or Axioms
- Duality Principle
  - the dual of an expression is found by replacing
    all instances of "$+$" with ".", all instances of "."
    with "$+$", all instances of "0" with "1" and
    all instances of "1" with "0"
  - if $A = B$ then $A^D = B^D$
- Proof by Contradiction

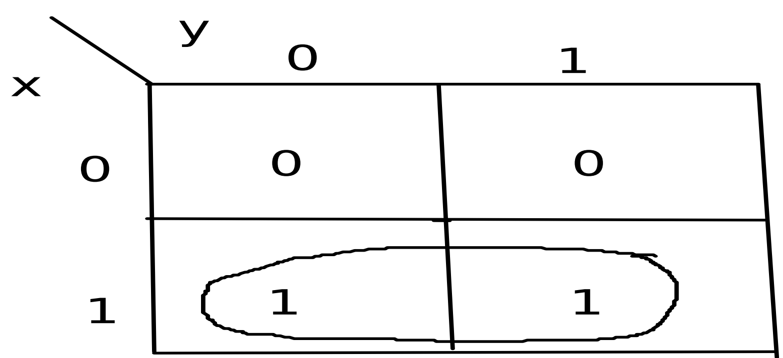
Theorems

- Idempotency
  - $x + x = x \quad x.x = x$
- Adsorption
  - $x.y + x = x \quad (x + y).x = x$
- Involution
  - $\overline{\overline{x}} = x$
- DeMorgans
  - $\overline{x.y} = \overline{x} + \overline{y}$
    $\overline{x + y} = \overline{x}.\overline{y}$
- Adjacency
  - $(x.y) + (x.\overline{y}) = x$
  - $(x + y).(x + \overline{y}) = x$

Adjacency Theorem

- Proof

$$(x.y) + (x.\overline{y}) = x$$
$$(x.y) + (x.\overline{y}) = x.(y + \overline{y})$$ (4) Distributivity
$$= x.(1)$$ (5) Complement
$$= x$$ (2) Identity



Other Logic Operators

x	y									Nor						Nand	
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
			And					Xor	Or		Xnor						