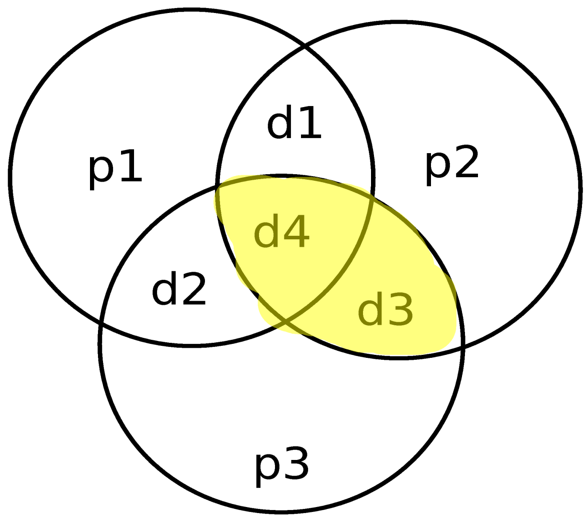
Computer Science CSCI 355

Digital Logic and Computer Organization

Dr. Peter Walsh
Department of Computer Science
Vancouver Island University
peter.walsh@viu.ca

Hamming Code

- $n = 4$ $m = 3$



e.g. c3 =1, c2=1, c1=0
d3 is in error

Hamming Code cont.

Parity bits cover sequences of data bits.

- $p1 = d1 \bigoplus d2 \bigoplus d4$
- $p2 = d1 \bigoplus d3 \bigoplus d4$
- $p3 = d2 \bigoplus d3 \bigoplus d4$

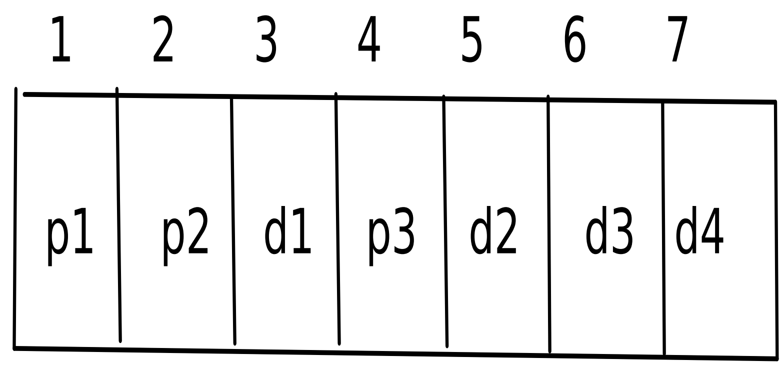
No data bits are covered by the same sequence of parity bits.

- $d1$ is covered by $p1, p2$
- $d2$ is covered by $p1, p3$
- $d3$ is covered by $p2, p3$
- $d4$ is covered by $p1, p2, p3$

Hamming Code cont.

Syndrome c3	c2	c1	Inference
0	0	0	no error
0	0	1	p1 in error
0	1	0	p2 in error
0	1	1	d1 in error
1	0	0	p3 in error
1	0	1	d2 in error
1	1	0	d3 in error
1	1	1	d4 in error

Transmit Bit Ordering



Example

- $d1, d2, d3, d4 = 0110$

1	2	3	4	5	6	7
p1	p2	d1	p3	d2	d3	d4
?	?	0	?	1	1	0

Example cont.

- $p1 = d1 \bigoplus d2 \bigoplus d4$
  $\Rightarrow p1 = 0 \bigoplus 1 \bigoplus 0 = 1$

- $p2 = d1 \bigoplus d3 \bigoplus d4$
  $\Rightarrow p2 = 0 \bigoplus 1 \bigoplus 0 = 1$

- $p3 = d2 \bigoplus d3 \bigoplus d4$
  $\Rightarrow p3 = 1 \bigoplus 1 \bigoplus 0 = 0$

Example cont.

- Transmit

1	2	3	4	5	6	7
p1	p2	d1	p3	d2	d3	d4
1	1	0	0	1	1	0

Example cont.

- Receive

1	2	3	4	5	6	7
p1	p2	d1	p3	d2	d3	d4
1	1	0	0	1	1	0

Example cont.

- $c1 = p1 \bigoplus d1 \bigoplus d2 \bigoplus d4$
  $\Rightarrow c1 = 1 \bigoplus 0 \bigoplus 1 \bigoplus 0 = 0$

- $c2 = p2 \bigoplus d1 \bigoplus d3 \bigoplus d4$
  $\Rightarrow c2 = 1 \bigoplus 0 \bigoplus 1 \bigoplus 0 = 0$

- $c3 = p3 \bigoplus d2 \bigoplus d3 \bigoplus d4$
  $\Rightarrow c3 = 0 \bigoplus 1 \bigoplus 1 \bigoplus 0 = 0$

- Syndrome = 000 $\Rightarrow$ no bits in error

Example cont.

- Receive

1	2	3	4	5	6	7
p1	p2	d1	p3	d2	d3	d4
1	1	1	0	1	1	0

Example cont.

- $c1 = p1 \oplus d1 \oplus d2 \oplus d4$
  $\Rightarrow c1 = 1 \oplus 1 \oplus 1 \oplus 0 = 1$

- $c2 = p2 \oplus d1 \oplus d3 \oplus d4$
  $\Rightarrow c2 = 1 \oplus 1 \oplus 1 \oplus 0 = 1$

- $c3 = p3 \oplus d2 \oplus d3 \oplus d4$
  $\Rightarrow c3 = 0 \oplus 1 \oplus 1 \oplus 0 = 0$

- Syndrome = 011 $\Rightarrow$ bit 3 in error = $d1$

Syndrome Size

- n data bits m parity bits
- range of $m >= n + m$
- $2^m - 1 >= n + m$
- e.g. $m = 4$ can protect a max of 11 data bits

Hamming Code cont.

Syndrome (c4,c3,c2,c1)	Inference
0 0 0 0	no error
0 0 0 1	p1 in error
0 0 1 0	p2 in error
0 0 1 1	d1 in error
0 1 0 0	p3 in error
0 1 0 1	d2 in error
0 1 1 0	d3 in error
0 1 1 1	d4 in error
1 0 0 0	p4 in error
1 0 0 1	d5 in error
1 0 1 0	d6 in error
1 0 1 1	d7 in error
1 1 0 0	d8 in error
1 1 0 1	d9 in error
1 1 1 0	d10 in error
1 1 1 1	d11 in error

Extended Hamming Code

- single bit error correction
- double bit error detection
- e.g. $n = 4$ add an extra parity bit $p4$ where

$$p4 = p1 \oplus p2 \oplus d1 \oplus p3 \oplus d2 \oplus d3 \oplus d4$$

$$c4 = p4 \oplus p1 \oplus p2 \oplus d1 \oplus p3 \oplus d2 \oplus d3 \oplus d4$$

Syndrome and $c4$ Inference

- $c4 = 0$ and syndrome $= 0 \Rightarrow$ no error
- $c4 \neq 0$ and syndrome $\neq 0 \Rightarrow$ single bit error (can be corrected)
- $c4 = 0$ and syndrome $\neq 0 \Rightarrow$ double bit error (can not be corrected, detection only)
- $c4 \neq 0$ and syndrome $= 0 \Rightarrow p4$ in error

Burst Errors

- form Hamming blocks on the "vertical" rather than the "horizontal"